Formal Concept Analysis 101 Order and Knowledge

Tom Hanika¹ & Robert Jäschke²

Knowledge & Data Engineering Group, University of Kassel
Berlin School of Library and Information Science, Humboldt-Universität zu Berlin

September 20, 2022

DIHMA.LAB/MaRDI "Digital Humanities meet Mathematics"

Order is All Around



Image sources: https://de.wikipedia.org/wiki/Zahlengerade Venita Oberholster via pixabay.com Own Screenshot Naeda Ehlers via pexels.com

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Mathematics Meets Humanities | Zuse Institute Berlin (ZIB)

Mathematics Meets Humanities Examples from ancient studies and psychology For many years, the humanities and mathematics were considered as two sciences that have little to do with one another. During the last decade, however, researchers on both sides realized the potential lying in the respective other field.

Https://pll.harvard.edu > course > introduction-digital-humanities

Introduction to Digital Humanities | Harvard University

Course outline. Digital Humanities and Data. Explain the term "digital humanities," and how it is understood across humanities disciplines. Describe the research journey as a partnership between researcher and library collections and staff. List examples of the limits of classification. Describe the implicit and explicit hierarchies that are ...

W https://en.wikipedia.org > wiki > Digital_humanities

Digital humanities - Wikipedia

Digital humanities (DH) is an area of scholarly activity at the intersection of computing or digital technologies and the disciplines of the humanities. It includes the systematic use of digital resources in the humanities, as well as the analysis of their application.

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CAST IN ORDER O MODERN	OF CHARACTERS F APPEARANCE IN THIS ABRIDGED VERSION
NARRATOR	Michael Benthall
IAGO	RALPH RICHARDSON
OTHELLO	JOHN GIELGUD
CASSIO	John Humphry
BRABANTIO	Ernest Hare
DUKE OF VEN	ICE Charles West
DESDEMONA	Barbara Jefford
EMILIA	Coral Browne
LODOVICO	Joss Ackland
emurone 1	Joss Ackland
SENATORS	David Tudor-Jones
SAILOR	Stephen Moore
	(Roger Grainger
SMALL PARTS	Peter Ellis
	David Lloyd-Meredith

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What is an order anyway?

Definition 1.1 (Ordered Set)

A set *X* together with a binary relation

 $\leq \, \subseteq X \times X$

is called (*partially*) ordered set, if for all $x, y, z \in X$ we find:

- $x \le x$ (reflexivity)
- if $x \le y$ and $y \le x \Rightarrow x = y$ (antisymmetry)
- if $x \le y$ und $y \le z \Rightarrow x \le z$ (transitivity)

Linear/total orders require that for all $x, y \in X$: $x \le y$ or $y \le x$ is true.

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Archaeological Stratigraphy (De Roo et al. 2016)



More Examples for non-linear orders



Project Schedule



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ge source:Gantt-Chart: own depictio

Learning to Order Data

Rembrandt's Paintings in Rijksm. (Wille 1992)



Binary Data Matrix

K	Fam.P.	Group.P.	Oak	Canvas	≥ 1660
Night		×		×	
Anat.		×		×	×
P. Titus	×			×	×
Staalm.				×	×
Mother	×		×		

Rembrandt's Paintings in Rijksm. (Wille 1992)



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Staalm.				×	×
Mother	×		×		



Larger Example: Characters and Scenes in Goethe's play "Egmont"



code: https://hu.berlin/notebooks

data source: https://dracor.org/ger/goethe-egmont

We can offer ...

- ... research methods that focus on *ordinal structures*.
- ... deriving *ontological and hierarchical knowledge* from data.
- ... procedures to uncover insights into *feature dependencies*.
- ... novel means for *temporal and spatial representation* of semantic features in your data.
- ... approaches that emphasize on *human-explainability* by design.

- ... talk with you about *your data*, its *features* and their *levels of measurement*.
- ... understand *your research questions* with respect to your data.
- ... talk about your requirements and expectations for *visual knowledge representation*.
- ... discuss with you *hybrid learning schemes* that may be applied in your research process.

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Thank you

- [De +16] Berdien De Roo et al. "Spatiotemporal data as the foundation of an archaeological stratigraphy extraction and management system." In: *Journal of Cultural Heritage* 19 (2016), pp. 522–530.
- [GW99] B. Ganter and R. Wille. *Formal Concept Analysis: Mathematical Foundations*. Springer-Verlag, Berlin, 1999, pp. x+284.
- [Wil92] R. Wille. "Concept lattices and conceptual knowledge systems." In: *Computers and Mathematics with Applications* 23 (1992), pp. 493–515.